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Abstract
It is expected that the implementation of minimal length in quantum models
leads to a consequent lowering of Planck's scale. In this paper, using the
quantum model with minimal length of Kempf *et al* (1994 *J. Math. Phys.* **35**
4483; 1995 *Phys. Rev.* D **52** 1108; 1997 *J. Phys.* A **30** 2093; 1996 *J. Math.
Phys.* **37** 2121), we examine the effect of the minimal length on the Casimir
force between parallel plates.

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1. Introduction

The construction of a quantized theory which incorporates gravity remains one of the priorities
of theoretical physicists. Unfortunately, all the attempts towards this goal fail. The reason
is that the Planck scale $l_\mathrm{p} = 1.616\,05 \times 10^{-35}$ m, at which the effects of quantum gravity
reveal themselves, is so small that these effects are neglected in experimentally accessible
energies. Recently, to solve this problem, different scenarios have been proposed, all leading
to a significant lowering of Planck's scale. Among them are the models with large extra
dimensions (LXD) [1], non-commutative field theory models [2] and models with nonzero
minimal lengths [3]. In this paper, we are interested in the later models based on generalized
commutation relations $[\hat{x}_i, \hat{p}_j] = i\hbar[(1 + \beta \hat{p}^2)\delta_{ij} + \beta' \hat{p}_i \hat{p}_j]$. These commutation relations
lead to a generalized uncertainty principle (GUP) which defines nonzero minimal lengths in
position and/or momentum. A nonzero minimal length in position has first appeared in the
context of perturbative string theory [4]. One major feature of this finding is that the physics
below such a scale becomes inaccessible and then defines a natural cut-off which prevents
the usual UV divergencies. The other consequence of such a GUP is the appearance of an
intriguing UV/IR mixing, first noticed in the ADS/CFT correspondence [5]. Physically, the
UV/IR mixing means that we can probe short distances physics by long distances physics. We
point out that the UV/IR mixing is also a feature of non-commutative quantum field theory
[2, 6]. On the other hand, some scenarios have been proposed where nonzero minimal length

is related to large extra dimensions [7], to the running coupling constant [8] and to the physics of black holes production [9].

Recently the cosmological constant problem and the classical limit of the physics with minimal length have been investigated by the group Virginia Tech [10, 11]. In [11] the value of the minimal length is so small that it seems meaningful. The size of the minimal length has also been extracted from the energy spectrum of the Coulomb potential [12, 13] and from the energy spectrum of electrons in a trap [14].

On the other hand, the Casimir force has been calculated in a model incorporating one large extra dimension [15]. A comparison with available experimental data gives $R \lesssim 10$ nm where R is the size of the compactified extra dimension. Motivated by the fact that large extra dimensions and minimal lengths models aim to lower Planck's scale and can be related to each other, we calculate, in this paper, the effect of the presence of a minimal length on the Casimir force between parallel plates.

The rest of the paper is organized as follows. In section 2, implementing the minimal length using standard methods of quantum mechanics we obtain the generalized uncertainty principle (GUP), generalized plane waves and modified closure relations. In section 3, we quantify the electromagnetic field and then following the standard recipe we calculate the Casimir force between two parallel plates. Section 4 is left for concluding remarks.

2. Quantum mechanics with generalized Heisenberg relation

Following [3], we consider the following realization of the position and momentum operators

$$X_i = i\hbar \left[(1 + \beta \mathbf{p}^2) \frac{\partial}{\partial p_i} \right], \qquad P_i = p_i, \tag{1}$$

where β is a small positive parameter. This representation leads to the following generalized commutators

$$[X_i, P_j] = i\hbar \delta_{ij}(1 + \beta \mathbf{p}^2), \tag{2}$$

$$[X_i, X_j] = 2i\hbar \beta(x_i p_j - p_i x_j), \tag{3}$$

$$[P_i, P_j] = 0 \tag{4}$$

and the generalized uncertainty principle (GUP)

$$(\Delta X_i)(\Delta P_i) \geqslant \frac{\hbar}{2}[1 + \beta(\Delta \mathbf{p})^2]. \tag{5}$$

The peculiarity of relation (5) is that it exhibits the UV/IR mixing phenomenon which allows us to probe short distance physics (UV) from long distance one (IR). A minimization of (5) with respect to (ΔP_i) gives the following nonzero minimal length

$$(\Delta X_i)_{\min} = \hbar \sqrt{\beta}. \tag{6}$$

Equation (6), like the UV/IR mixing, reveals the non-local character of models based on equations (1)–(3). Then we have not localized eigenfunctions in the $\mathbf{r}$-space. So, any eigenvalue problem can be solved by going to the momentum space.

In the following, we derive necessary relations for our calculation taking in mind that we must recover the usual quantum mechanics in the limit $\beta \to 0$. First, we assume that $\mathbf{R}|\mathbf{r}\rangle = \mathbf{r}|\mathbf{r}\rangle$ where the vectors $|\mathbf{r}\rangle$ represent maximally localized states. They are normalized states unlike the ones of ordinary quantum mechanics.

Using these maximally localized states we derive the following quasi-position eigenvectors

$$f_{\mathbf{p}}(\mathbf{r}) = \frac{1}{\sqrt[3]{2\pi\hbar}} \exp\left(-\frac{i\mathbf{r}}{\hbar\sqrt{\beta}} \arctan \mathbf{p}\sqrt{\beta}\right) \tag{7}$$

with the following generalized dispersion relation

$$\lambda(|\mathbf{p}|) = \frac{2\pi\hbar\sqrt{\beta}}{\arctan(|\mathbf{p}|\sqrt{\beta})}. \tag{8}$$

The states given by (7) are far from being the well-known plane waves. However, in the limit $\beta \to 0$ we recover the usual plane waves of ordinary quantum mechanics.

Now assuming the usual closure relation for the maximally localized eigenstates $1 = \int_{-\infty}^{+\infty} d\mathbf{r}' |\mathbf{r}\rangle\langle\mathbf{r}|$, we obtain

$$\langle\mathbf{p}' \mid \mathbf{p}\rangle = \sqrt{\beta}\delta(\arctan\sqrt{\beta}\mathbf{p} - \arctan\sqrt{\beta}\mathbf{p}'). \tag{9}$$

Using the relation $\delta f(x) = \Sigma_i \frac{\delta(x-x_i)}{f'(x_i)}$, where x_i are the roots of $f(x)$, we finally get

$$\langle\mathbf{p}' \mid \mathbf{p}\rangle = (1 + \beta\mathbf{p}^2)^{\frac{1}{2}}(1 + \beta\mathbf{p}'^2)^{\frac{1}{2}}\delta(\mathbf{p} - \mathbf{p}'). \tag{10}$$

From this equation we derive the modified completeness relation for the momentum eigenstates $|\mathbf{p}\rangle$

$$\int \frac{d\mathbf{p}}{(1 + \beta\mathbf{p}^2)} |\mathbf{p}\rangle\langle\mathbf{p}| = \mathbf{1}. \tag{11}$$

Here we observe a squeezing of the momentum space at high momentum. Let us end our calculations by showing that the states $|\mathbf{r}\rangle$, like the coherent states, do not form an orthogonal set. Indeed, we have

$$\begin{aligned}
\langle\mathbf{r} \mid \mathbf{r}'\rangle &= \int \frac{d\mathbf{p}}{(1 + \beta\mathbf{p}^2)} f_{\mathbf{p}}(\mathbf{r}) f_{\mathbf{p}}^*(\mathbf{r}') \\
&= \int \frac{d\mathbf{p}}{(1 + \beta\mathbf{p}^2)} \exp\left\{-\frac{i(\mathbf{r} - \mathbf{r}')}{\hbar\sqrt{\beta}} \arctan\sqrt{\beta}\mathbf{p}\right\} \\
&= \frac{1}{\pi(\mathbf{r} - \mathbf{r}')} \sin\left(\frac{\pi(\mathbf{r} - \mathbf{r}')}{2\hbar\sqrt{\beta}}\right).
\end{aligned} \tag{12}$$

The right-hand side is a well-behaved function unlike the Dirac distribution of ordinary quantum mechanics. It is clear that the limit $\beta \longrightarrow 0$ restores the usual normalization $\langle\mathbf{r} \mid \mathbf{r}'\rangle = \delta(\mathbf{r} - \mathbf{r}')$. In conclusion, we have chosen to work with the normalization constant $1/\sqrt[3]{2\pi\hbar}$, while this choice renders the states given by equation (7) unphysical, to reproduce in the limit $\beta \to 0$ the usual results of quantum mechanics.

3. Casimir effect

The most general solution of the Maxwell equations in the presence of a minimal length in the Coulomb gauge for slowly moving particles is given by

$$\hat{\mathbf{A}}(\mathbf{r}, t) = \sqrt{2\pi\hbar^3 c\sqrt{\beta}} \int \frac{d\mathbf{p}}{(1 + \beta\mathbf{p}^2)\sqrt{\arctan\sqrt{\beta}|\mathbf{p}|}} \sum_{\gamma=\pm 1} \left[f_\gamma(\mathbf{p}, \omega)\hat{a}_\gamma(\mathbf{p}) + f^*(\mathbf{p}, \omega)\hat{a}_\gamma^\dagger(\mathbf{p})\right] \tag{13}$$

where $f_\gamma(\mathbf{p}, \omega)$ are generalized plane waves which can be obtained from equation (7),

$$f_\gamma(\mathbf{p}, \omega) = \frac{\varepsilon_\gamma(\mathbf{p})}{\sqrt[3]{2\pi\hbar}} \exp\left(\frac{i}{\hbar\sqrt{\beta}}[\mathbf{r}\arctan(\mathbf{p}\sqrt{\beta}) - \hbar\omega(|\mathbf{p}|)t]\right),\tag{14}$$

with $\omega(|\mathbf{p}|)$ defined by the generalized dispersion relation (8) and $\varepsilon_\gamma(\mathbf{k})$ are the polarization vectors verifying

$$\varepsilon_\gamma(\mathbf{p})\varepsilon_{\gamma'}^*(\mathbf{p}) = \delta_{\gamma\gamma'}.\tag{15}$$

From (14) we derive the following normalization condition:

$$\int d\mathbf{r}\, f_\gamma^*(\mathbf{p}, \omega)i\overleftrightarrow{\partial_0}f_{\gamma'}(\mathbf{p}', \omega') = \delta_{\gamma\gamma'}(1 + \beta\mathbf{p}^2)^{\frac{1}{2}}(1 + \beta\mathbf{p}'^2)^{\frac{1}{2}}\delta(\mathbf{p} - \mathbf{p}').\tag{16}$$

The creation and annihilation operators are non-relativistic ones and, since the momentum operators are commuting, they satisfy the usual commutation relation,

$$[\hat{a}_\gamma(\mathbf{p}), \hat{a}_{\gamma'}^\dagger(\mathbf{p}')] = \delta_{\gamma\gamma'}\delta(\mathbf{p} - \mathbf{p}').\tag{17}$$

This result, along with equation (12), can be used to derive a modified commutation relation between the fields:

$$[A^i(\mathbf{r}, t), E^j(\mathbf{r}', t)] = i\left(\delta^{ij} - \frac{\partial^i\partial^j}{\nabla^2}\right)\frac{\sin\left(\frac{\pi(\mathbf{r}-\mathbf{r}')}{2\hbar\sqrt{\beta}}\right)}{\pi(\mathbf{r} - \mathbf{r}')}.\tag{18}$$

Using the well-known relation $\delta(x) = \lim_{\varepsilon\to 0}\frac{\sin(x/\varepsilon)}{\pi x}$, we obtain the usual commutation relation in the limit $\beta \to 0$.

Armed with this background, let us then attack the Casimir effect with square parallel plates of sides L. Then the electromagnetic field must satisfy boundary conditions. In our case, we have from (14)

$$\kappa_3 = \frac{\hbar n\pi}{a},\tag{19}$$

where a is the plates separation, $\kappa_3 = \frac{p_3}{\sqrt{\beta(\mathbf{q}^2 + p_3^2)}}\arctan\left(\sqrt{\beta(\mathbf{q}^2 + p_3^2)}\right)$ and $\mathbf{q}$ is the transverse momentum along the plates. In (19) we have a finite number of modes $n = 0, 1, 2, \ldots, n_{max} = \left[\frac{a}{2(\Delta x)_{min}}\right]$ where $[\cdots]$ denotes the next smaller integer. Then the geometrical quantization given by (19) fulfils the requirement that in quantum models with a minimal length, the Compton wavelength cannot take arbitrary values. Indeed we have $\lambda_{min} = 4\hbar\sqrt{\beta}$.

Since β is a small parameter, we have tried a series solution to the eighth order in β. In the following, we just show the following truncated solution:

$$p_3(n) = \frac{\hbar n\pi}{a}\left[1 + \frac{\beta}{3}\left(\mathbf{q}^2 + \left(\frac{\hbar n\pi}{a}\right)^2\right) + \frac{\beta^2}{45}\left(2\left(\mathbf{q}\frac{\hbar n\pi}{a}\right)^2 - 4\mathbf{q}^4 + 12\left(\frac{\hbar n\pi}{a}\right)^4\right) + \cdots\right].\tag{20}$$

In figure 1 we have plotted the modified wavelengths associated with momentums κ_3 and p_3 to the eighth order in β for $\beta = 0.01$ and $\hbar = a = q = 1$. For large n the wavelength associated with κ_3 tends asymptotically to λ_{min} while the one associated with p_3 tends to zero faster than the wavelength of the usual theory. A similar behaviour has been obtained in [16] using generalized dispersion relations.

The potential vector in the presence of the plates is then given by

$$\hat{\mathbf{A}}_a(\mathbf{r}, t) = \sqrt{2\pi\hbar^3 c\sqrt{\beta}}\frac{\hbar\pi}{a}\sum_{\substack{n=-n_{max}\\\gamma=\pm 1}}^{n_{max}}\int\frac{d\mathbf{q}}{(1 + \beta\mathbf{p}^2(a))\sqrt{\arctan\sqrt{\beta}|\mathbf{p}(a)|}}$$

$$\times\left\{f_\gamma(\mathbf{p}(a), \omega)\hat{a}_\gamma(\mathbf{p}(a)) + f^*(\mathbf{p}(a), \omega)\hat{a}_\gamma^\dagger(\mathbf{p}(a))\right\},\tag{21}$$
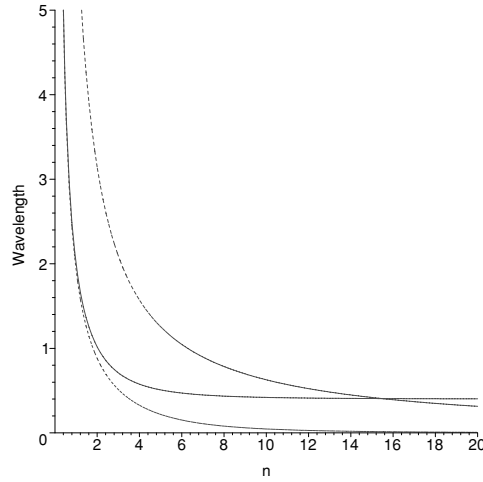
Figure 1. Plot of Compton wavelengths associated with the momentums κ_3 (solid), p_3 (dot) for $\beta = 0.01$ and the usual one (dash–dot) versus the quantum number n.

where

$$\mathbf{p}(a) = \mathbf{q} + \mathbf{p}_3(n). \tag{22}$$

The commutation relation between the creation and annihilation operators is then affected by the solution (20). For our purpose it suffices to use the following approximation

$$\left[\hat{a}_\gamma(\mathbf{p}(a)), \hat{a}^\dagger_{\gamma'}(\mathbf{p}'(a))\right] \simeq \frac{a}{\pi\hbar} \delta_{nn'} \delta_{\gamma\gamma'} \delta(\mathbf{q} - \mathbf{q}') + O(\beta). \tag{23}$$

The energy shift resulting from the presence of the plates is defined by the relation

$$\Delta E = \langle 0|(\hat{H}(a) - \hat{H})|0\rangle$$

$$= \frac{1}{8\pi} \int d\mathbf{r} \langle 0|\{(\partial_0\hat{\mathbf{A}}_a)^2 - \hat{\mathbf{A}}_a\Delta\hat{\mathbf{A}}_a + (\partial_0\hat{\mathbf{A}})^2 - \hat{\mathbf{A}}\Delta\hat{\mathbf{A}}\}|0\rangle. \tag{24}$$

Performing the standard calculation we get

$$\Delta E = \frac{cL^2}{8\pi\hbar^2\beta^{\frac{1}{2}}} \int d\mathbf{q} \left\{ \sum_{n=-n_{\max}}^{n_{\max}} \frac{\arctan\sqrt{\beta\left(\mathbf{q}^2 + p_3^2(n)\right)}}{1 + \beta\left(\mathbf{q}^2 + p_3^2(n)\right)} \right.$$

$$\left. - \int_{-\nu_{\max}}^{\nu_{\max}} d\nu \frac{\arctan\sqrt{\beta\left(\mathbf{q}^2 + p_3^2(\nu)\right)}}{1 + \beta\left(\mathbf{q}^2 + p_3^2(\nu)\right)} \right\}. \tag{25}$$

From this expression it is easily seen that the terms proportional to $\beta^{n\geqslant 1}$ in $p_3(n)$ and the omitted terms in the commutation relation (23) will give negligible contributions proportional to $\beta^{n\geqslant 2}$.

Exchanging sums and integrals and defining the following quantity

$$G(\nu) = \frac{1}{\sqrt{\beta}} \int_0^\infty dx \frac{\arctan\sqrt{\beta\left(p_3^2(\nu) + x\right)}}{1 + \beta\left(p_3^2(\nu) + x\right)}, \tag{26}$$

the energy shift per unit area $\Delta\mathcal{E} = \frac{\Delta E}{L^2}$ is given by

$$\Delta\mathcal{E} = \frac{c}{4\pi\hbar^2}\left\{\sum_{n=0}^{n_{\max}} G(n) - \int_0^{\nu_{\max}} \mathrm{d}\nu\, G(\nu) - \frac{1}{2}G(0)\right\}. \tag{27}$$

With the aid of the variable $\rho = \frac{1}{\sqrt{\beta}}\arctan\sqrt{\beta\left(x + p_3^2(\nu)\right)}$, the function $G(\nu)$ is simply given by

$$G(\nu) = \frac{2}{\sqrt{\beta}}\int_{\frac{1}{\sqrt{\beta}}\arctan p_3(\nu)\sqrt{\beta}}^{\frac{\pi}{2\sqrt{\beta}}} \tan(\sqrt{\beta}\rho)\rho\,\mathrm{d}\rho. \tag{28}$$

Using the following expansion [18]

$$t\tan t = \sum_{k=1}^{\infty} \frac{2^{2k}(2^{2k}-1)B_k}{(2k)!}t^{2k}, \qquad |t| < \frac{\pi}{2} \tag{29}$$

and performing the integral over ρ we obtain

$$G(\nu) = \sum_{k=1}^{\infty} \frac{\beta^{k-1}2^{2k+1}(2^{2k}-1)B_k}{(2k+1)(2k)!}\left[\left(\frac{\pi}{2\sqrt{\beta}}\right)^{2k+1} - \left(\frac{1}{\sqrt{\beta}}\arctan p_3(\nu)\sqrt{\beta}\right)^{2k+1}\right] \tag{30}$$

where B_k are Bernoulli's numbers given by $B_1 = \frac{1}{6}$, $B_2 = \frac{1}{30}$, $B_3 = \frac{1}{42}, \dots$ [17].

It is important to note here that we have not introduced any cut-off as is the case in the ordinary Casimir effect. The cut-off $\frac{1}{\sqrt{\beta}}$ is implemented naturally in equation (28). In equation (30) the contributions for $n > n_{\max}$ are negligible compared to the ones for $n \leqslant n_{\max}$ since $\frac{1}{\sqrt{\beta}}\arctan p_3(\nu)\sqrt{\beta}$ tends asymptotically to $\frac{\pi}{2\sqrt{\beta}}$ for $n > n_{\max}$. For the rest of the calculation the first term is irrelevant for our purpose and we ignore it. Then we can extend the summation over n and ν in equation (27) from 0 to $+\infty$. Thus

$$\Delta\mathcal{E} = \frac{c}{4\pi\hbar^2}\left\{\sum_{n=0}^{\infty} G(n) - \int_0^{\infty} \mathrm{d}\nu\, G(\nu) - \frac{1}{2}G(0)\right\}. \tag{31}$$

At this stage we can use Euler's formula [17]

$$\int_0^{\infty} f(x)\,\mathrm{d}x = \sum_{n=0}^{\infty} f(n) - \frac{f(0)}{2} + \sum_{m=1}^{\infty} \frac{B_{2m}}{(2m)!}f^{(2m-1)}(0) \tag{32}$$

to obtain

$$\Delta\mathcal{E} = -\frac{c}{4\pi\hbar^2}\sum_{m=1}^{\infty} \frac{B_{2m}}{(2m)!}G^{(2m-1)}(0) \tag{33}$$

where B_{2m} are the Bernoulli numbers and $G^{(l)}(0)$ are derivatives of $G(\nu)$ at $\nu = 0$.

Using the expression of $p_3(\nu)$ given by (20) in (30) we obtain a first-order expansion in β (recall that the commutation relations are valid to the first order in β)

$$G(\nu) = -4B_1\left(\frac{\hbar\pi\nu}{a}\right)^3 + 4\beta\left[\frac{B_1}{3} + B_2\right]\left(\frac{\hbar\pi\nu}{a}\right)^5. \tag{34}$$

Using $B_1 = \frac{1}{6}$, $B_2 = \frac{1}{30}$, we finally obtain

$$G(\nu) = -\frac{2}{3}\left(\frac{\hbar\nu\pi}{a}\right)^3 + \frac{48}{135}\beta\left(\frac{\hbar\nu\pi}{a}\right)^5. \tag{35}$$

Then from equation (33) we have

$$\Delta\mathcal{E} = -\frac{c}{4\pi\hbar^2}\left[\frac{B_4}{4!}G^{(3)}(0) + \frac{B_6}{6!}G^{(5)}(0)\right].$$ (36)

Evaluating the derivatives at $\nu = 0$ and using $B_4 = \frac{1}{30}$, $B_6 = \frac{691}{2730}$, we obtain

$$\Delta\mathcal{E} = \hbar c\left\{\frac{\pi^2}{720a^3} - \beta\frac{691}{284\,275}\frac{\hbar^2\pi^4}{a^5}\right\}.$$ (37)

The force per unit surface $\mathcal{F} = \frac{\partial}{\partial a}\Delta\mathcal{E}$ generated by this energy is given by

$$\mathcal{F} = -\hbar c\left\{\frac{1}{240}\frac{\pi^2}{a^4} - \beta\frac{691}{36\,855}\frac{\hbar^2\pi^4}{a^6}\right\}.$$ (38)

It is clear from this result that for a fixed separation of the plates, the Casimir force in the presence of a minimal length may be attractive or repulsive depending on the value of the minimal length $(\Delta x)_{\min} = \hbar\sqrt{\beta}$.

The first term in equation (38) is the standard attractive Casimir force [19] which alone is a source of instability. Indeed the two-plate systems can collapse to a one-plate system. The second term which is the correction arising from the presence of the minimal length is the repulsive contribution to the Casimir force and therefore provides the desired stability of the two-plate systems. This is important for the construction of consistent Kaluza–Klein theories. The same results have been obtained by [20] for the Casimir effect in κ-deformed theory and by [16] for a particular implementation of the minimal length.

The condition for a quantum stability of the two-plate systems gives the following constraint

$$\frac{(\Delta x)_{\min}}{a} \sim 0.15.$$ (39)

Using the experimentally accessible plates separations, which are of order 100 nm [22], we obtain

$$\hbar\sqrt{\beta} \sim 15\text{ nm}.$$ (40)

However, for the force to remain attractive, as is usually observed, we have the condition $\frac{\hbar\sqrt{\beta}}{a} \lesssim 0.15$.

Figure 2 illustrates the variation of the Casimir force for different values of minimal length. It is clear that this force becomes repulsive for $\frac{(\Delta x)_{\min}}{a} > 0.15$. Let us point out that in the plot a is always greater than $(\Delta x)_{\min}$ because the Casimir force for plate separation below the minimal length is meaningful since the space below this scale is fuzzy and then experimentally inaccessible.

Before ending this section we note that the Casimir force in the presence of one compactified extra dimension lies below the standard Casimir force [15], while in the case of a minimal length it lies above. Therefore we can conclude that the effects of the minimal length and the extra dimensions are opposites. This is expected from the beginning since the minimal length squeeze the momentum space at high momentum and then the natural cut-off of the model suppresses the contributions of such momentum. Finally, our treatment along with the work in [16] contradict the one in [21] where the Casimir force in the presence of a
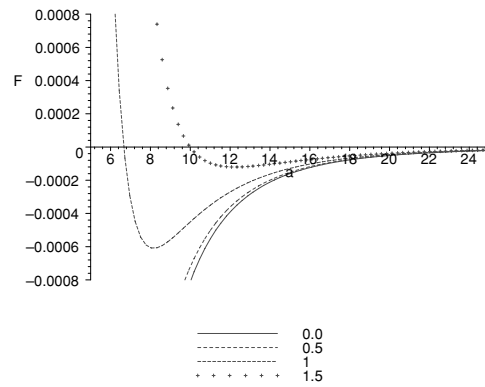
Figure 2. Plot of Casimir force F (eV nm^{-3}) versus the plates separation a (nm) for different values of the minimal length $\hbar\sqrt{\beta}$.

minimal length has been found to be a discontinuous function of the plate separation, a result essentially due to an appropriate geometric quantization between the plates.

4. Conclusion

In this paper, we considered the effect of minimal length on the Casimir force between parallel plates. We have shown that the minimal length acts like a natural cut-off which suppresses the contribution of unwanted high momentum. Using the accessible plates separation used for an experimental calculation of the Casimir force, we found an upper bound for the minimal length of the same order of the size of one compactified extra dimension. However, this bound is already excluded from high precision measurements and collider experiments [7] and then we recover the usual attractive character of the Casimir force. The Casimir force in the presence of minimal length in the context of a model with one extra dimension is under investigation and will be published elsewhere.

Acknowledgments

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